THE SENSITIVITY OF SOLAR EIGENFREQUENCIES TO THE TREATMENT OF THE EQUATION OF STATE

Roger K. Ulrich
Department of Astronomy, University of California, Los Angeles, CA 90024, U.S.A.

Edward J. Rhodes Jr.
Department of Astronomy and Earth and Space Science Institute, University of Southern California, Los Angeles, CA 90007 and Jet Propulsion Laboratory, Pasadena, CA 91109, California Institute of Technology, U.S.A.

Abstract: We have examined the sensitivity of solar eigenfrequencies to uncertainties in the equation of state. The principal uncertainties in the equation of state involve the treatment of pressure ionization, the Debye-Huckel coulomb corrections and the treatment of many-particle interaction effects. We find that for the lowest degree modes (ℓ between 0 and 3) the terms and procedures used in our equation of state which deal with these uncertainties introduce changes in the frequencies which are less than 4 μHz. Recently, Shibahashi, Noels and Gabriel (1983) published solar eigenfrequencies using a theory with an equation of state improved with respect to the theory used earlier by Shibahashi and Osaki (1981). Their comparison between the two sets of results suggested that uncertainties in the frequencies as large as 10 μHz could be caused by the equation of state. We feel that since the entire effect of the uncertain terms is only 4 μHz and since the uncertainties are only a fraction of each term, the 10 μHz changes found by Shibahashi et al must be a consequence of differences between the earlier and later calculations in areas other than the equation of state.

1. Introduction

Although the solar equation of state is largly uncomplicated, there remain uncertainties in the areas of pressure ionization, free particle scattering states and the effects of multi-particle interactions. The magnitude of the potential problem is made apparent by a simple application of Saha's equation to the solar center which shows that without any additional physical effects beyond simple ionization roughly 25% of the hydrogen should be neutral even though the temperature is 15 million degrees. In fact additional physical principles come into the problem and reduce the abundance of incompletely ionized species well below the level indicated by the Saha equation. A discussion of the proper approach to the equation of state is given elsewhere in this volume by Rogers (1984). We give here a very brief summary of the areas of uncertainty.

First, the effect due to the electrostatic interaction between the charged particles alters the pressure and energy density in a manner which is frequently treated by the Debye-Huckel approximation. The term which is added to the thermodynamic quantities due to this approximation is sometimes called the

Coulomb correction. The presence of the plasma charge surrounding each nucleus produces a screening effect which alters the bound electron energy levels so that they approach the continuum level and eventually the electron becomes unbound. Second, a cluster expansion (see Mayer 1958 and Eyring, Henderson, Stover and Eyring 1982 for discussions of the cluster expansion method) of the activity (the thermodynamic quantity which is like the pressure but which is most readily calculated from the density) in terms of the density shows that the bound states and the states of free electrons near nuclei (called scattering states) produce counteracting effects on the activity. To lowest order the combined result can be represented by using an effective internal partition function in the ionization equilibrium equation which replaces the exponential factors of the form exp(E/kT) in the Saha equation with the Planck-Larkin factor [exp(E/kT) - 1 - E/kT]. This factor has an asymptotic value of zero instead of unity when E/kT approaches zero. Third, the complications from many-body interactions are not eliminated by this form of the Saha equation; instead their effects are shifted from the formation of bound states to the alteration of higher order cluster terms in the description of the scattering states. Ebeling and Sandig (1973) have evaluated these terms for a gas of pure hydrogen and given their result in the form of a Pade approximation to the leading additional term in the cluster expansion - the 2nd Virial term.

We present here eigenfrequencies for solar models which were calculated to study the effects of the individual terms in the equation of state. These models were originally presented in the paper by Ulrich (1982). The treatment of the changes in the energy levels due to screening in that paper is not optimal according to the discussion by Rogers in this volume. Nonetheless, the effect of partial ionization was found to be entirely negligible as long as the Planck-Larkin form of the ionization equilibrium equation was used. The recent discussion by Rouse (1983) of the Planck-Larkin equation is based on the faulty assumption that the effective partition function is the sum of actual occupation numbers of the available energy levels. Although for a non-interacting gas this would be a valid assumption, in the case of the solar equation of state it is not. The compensation of the effects of the bound states by the scattering states represented by the Planck-Larkin equation results from the reduction of the number of unbound states available to free electrons because the wave functions of the scattering states are distorted by the presence of the nuclear charge. Rogers (1977) has given a discussion of this compensation in terms of a WKB description of the wave functions. In order to make comparisons with astrophysical data, the actual atomic occupation numbers must be calculated separately from a thermodynamically self-consistent formulation. Such a calculation has not yet been made.

2. The Models

The models which we use here were all computed within a few days of each other. During this period, the only quantities which were varied were the various equation of state terms. We have subsequently made other changes and improvements which have made the 'Standard Model' of the Ulrich (1982) paper obsolete. Nonetheless, this set of models is valuable because it provides an opportunity to study the equation of state effects in isolation. Because the Planck-Larkin form of the ionization equilibrium equation is well established, we have concentrated on the three models of the Ulrich (1982) paper which use that formulation to treat the ionization explicitly instead of artificially assuming full ionization. The three models are Model 19 which omits both the Coulomb correction and the scattering state collective effects, Model 21 which includes the Coulomb correction but omits the collective effects, and Model 22 which includes both effects. We find that the frequencies change by less than 4 μHz as a result of any of the equation of state terms. For comparison, we also computed frequencies for model 10 which incorrectly uses the Saha equation throughout the solar interior. This model includes such erroneous results as an abundance of neutral hydrogen at the solar center equal to 0.25 even though there would be inadequate volume for so many bound electrons without having adjacent wave functions overlap.

3. Theoretical Frequencies

We have computed frequencies of the solar oscillations using the methods described by Ulrich and Rhodes (1983). We adopt here a slightly different

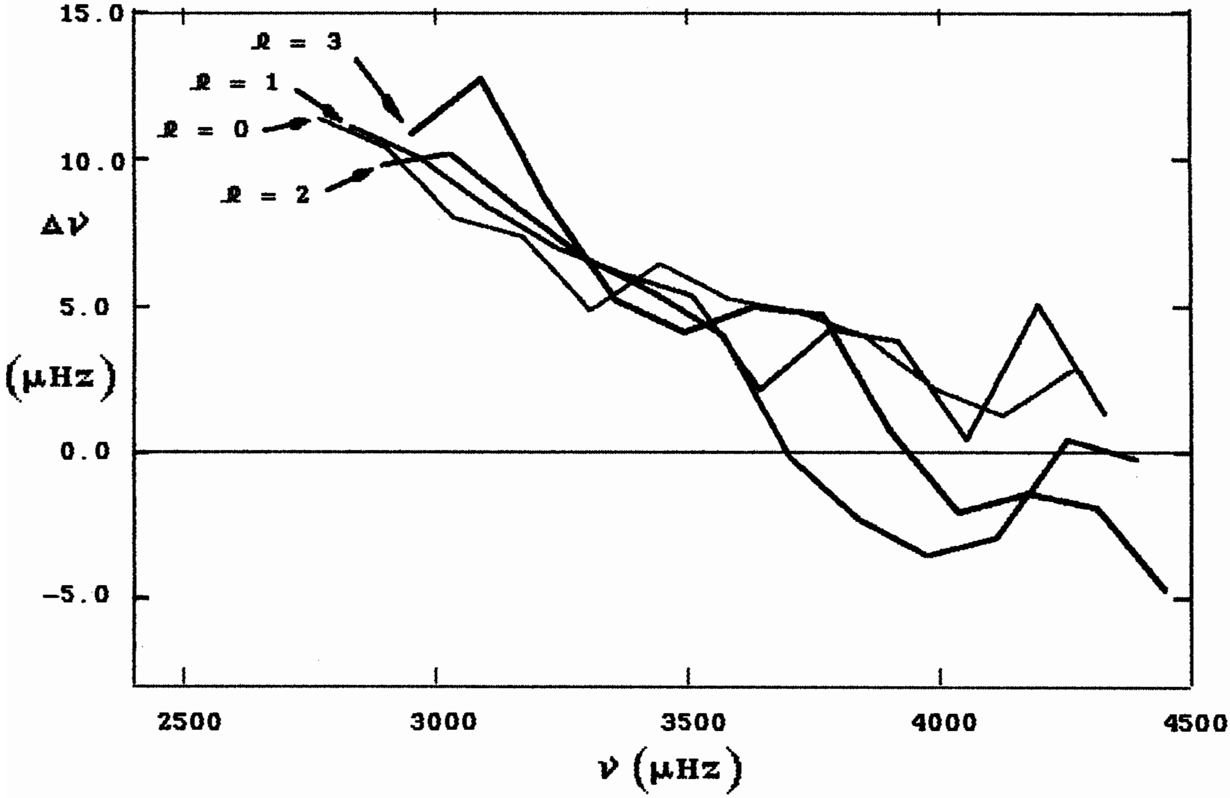

Figure 1 Comparison between the theoretical and observed frequencies. The plot shows the frequency differences: observed minus theoretical. The observations are from Grec, Fossat and Pomerantz (1983) and the theory is for the standard model which incorporates the Planck-Larkin ionization equation, the Coulomb corrections and the 2nd Virial term. The values of ℓ are indicated by the thickness of the lines with the thinnest having ℓ = 0 and the thickest having ℓ = 3.

format for the presentation of the results than we have used in the past. Rather than showing the individual frequencies on some form of folded plot, we show only differences between frequencies. Figure 1 shows the differences between the observations and the frequencies for the standard model. Each line in this figure shows the discrepancy as a function of the frequency for fixed values of ℓ. The erratic behavior at the higher frequencies in this figure and subsequent figures is caused by the presence in the theory of a resonant region in the chromosphere. When the interior mode frequencies and the chromospheric mode frequencies are nearly equal then the chromospheric mode can perturb the interior mode frequency. The existence of this perturbation enhances the sensitivity of the frequencies to changes in the models.

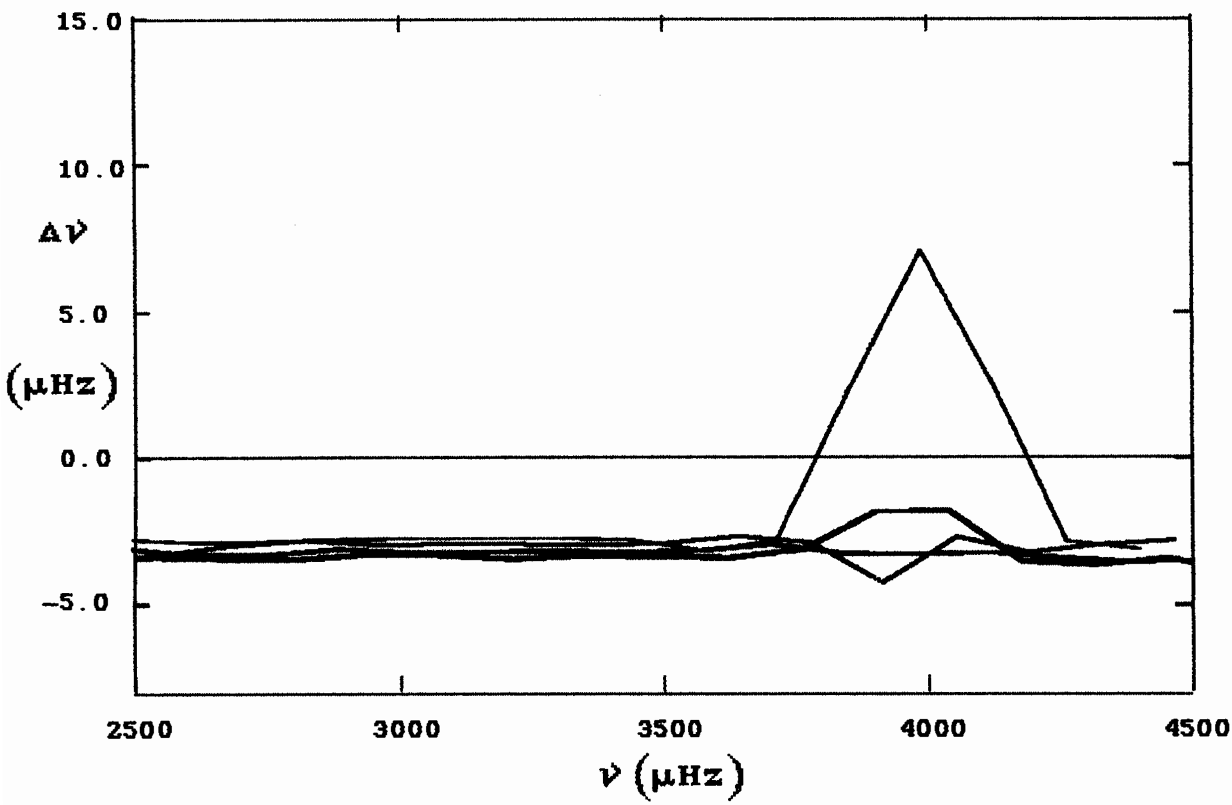

Figure 2 The effect of adding the collective effects as represented by the 2nd Virial term. Frequency differences for the model with the term minus the model without the term are plotted. The values of ℓ are indicated as in Figure 1.

Figures 2 and 3 show the changes in the oscillation frequencies due to the influence of the collective effects as treated in the form of the 2nd virial term and the influence of the Coulomb corrections. The frequency dependence of the effects are nearly independent of ℓ as long as ℓ is near 0. The eigenfunctions for these oscillations have very similar dependence on radius in the outer layers of the sun. Since the largest changes in the equation of state due to the collective effects and the Coulomb corrections occur in the outer

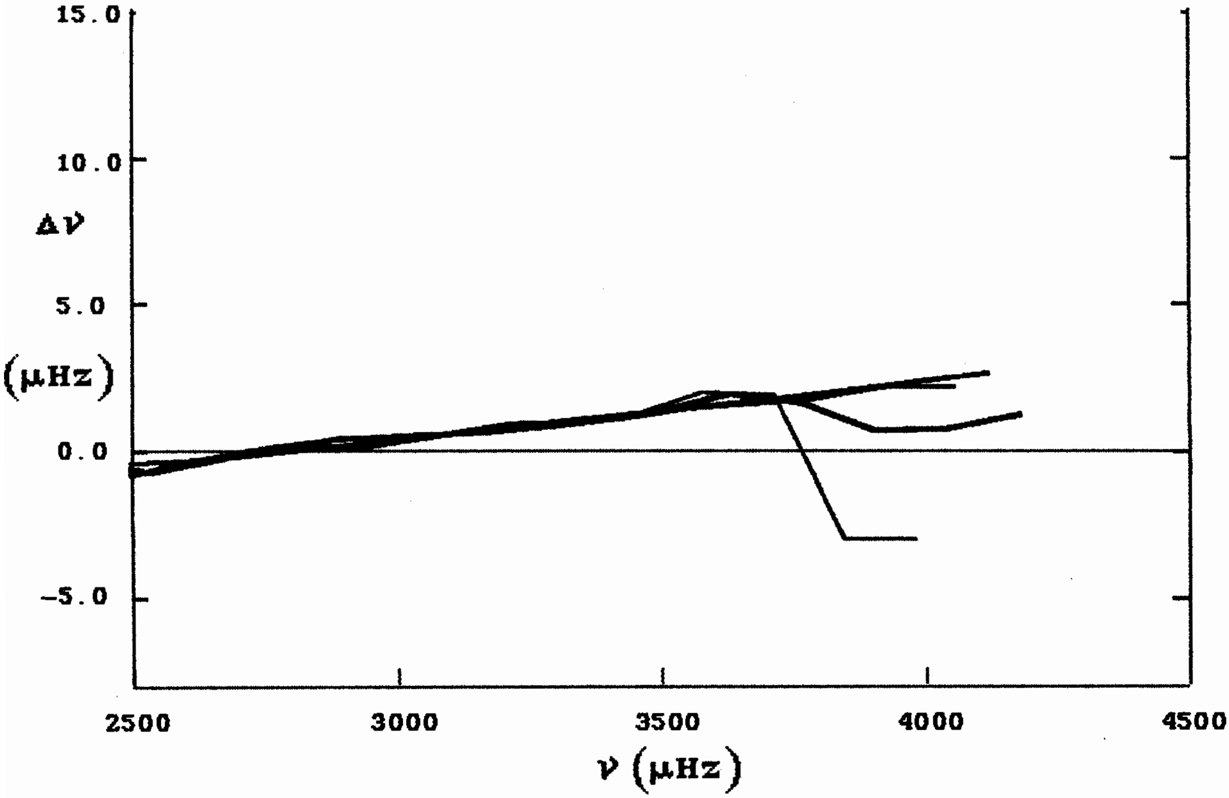

Figure 3 The effect of adding the Coulomb correction. Frequency differences for the model with the correction minus the model without the correction are plotted. The values of ℓ are indicated as in Figure 1.

sections of the sun, the actual value of ℓ has little impact on the frequencies unless ℓ is very much larger than 4. Both the magnitude and dependence on ν of the changes due to the coulomb corrections and the collective effects are inadequate to play a major role in explaining the discrepancy between theory and observation. One other equation of state procedure is of interest even though it is not really an area of uncertainty and that is the effect of using the Saha equation throughout the solar model. The results of this calculation are not to be taken seriously since the high abundance of neutral hydrogen predicted for the solar core with this equation is unphysical due to the scattering states and the excessive volume occupied by the bound electrons. Nonetheless, this equation of state is significantly different from the one used in the standard model and therefore gives some indication of an upper bound on the range of variation in the frequencies which might be caused by a major change in the equation of state. Figure 4 shows a comparison between two models which differ only in their treatment of the ionization equilibrium. Both models included the Coulomb corrections but the model using the Planck-Larkin equation omitted the 2nd Virial Term. The frequency changes shown in Figure 4 are larger than those in Figures 2 and 3 but still not large enough to explain the discrepancy between the theory and observation and the model with the Planck-Larkin equation is in slightly better agreement with observation than the model with the Saha equation. Note the wavy nature of the frequency changes. This pattern is caused by the fact that the largest changes between the two models are localized near the model center. As the eigenmode structures change with frequency, maxima and minima in the eigenfunctions move past this localized region and cause the frequency shifts to vary in response.

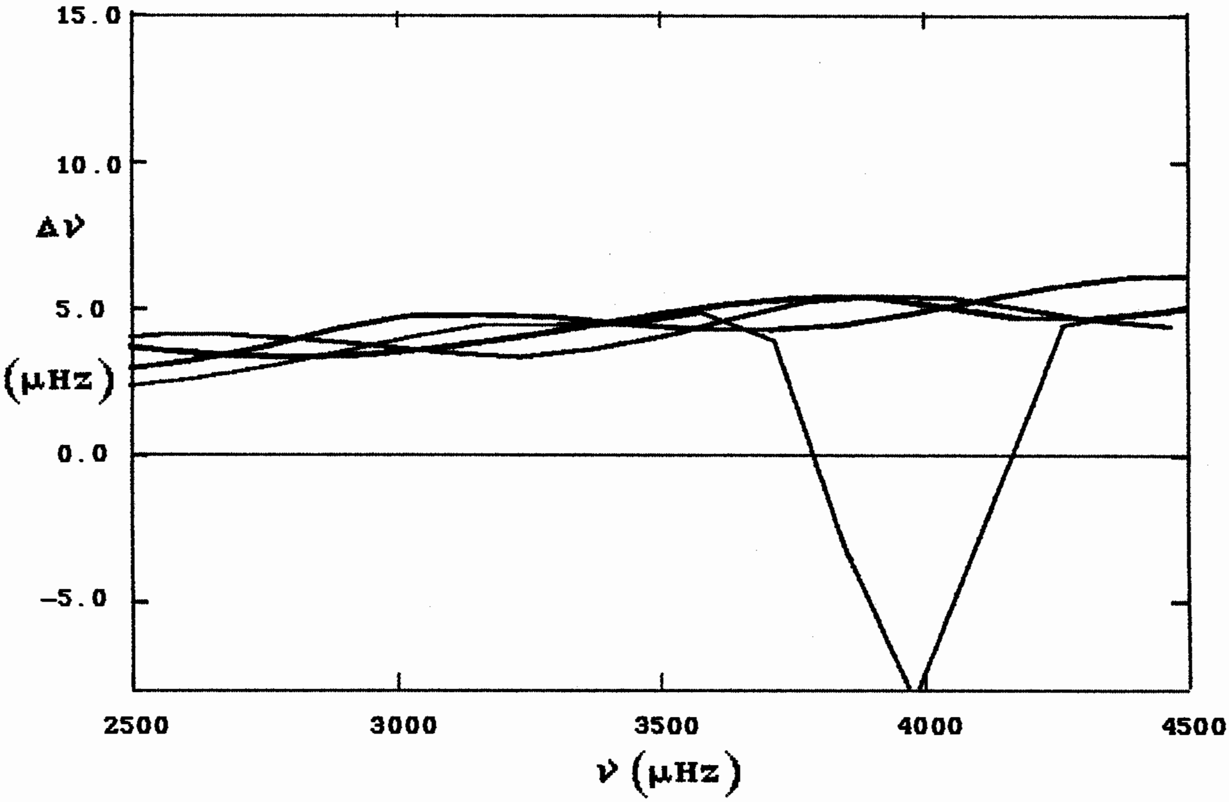

Figure 4 The effect of using the Saha equation for the ionization equilibrium instead of the Planck-Larkin equation. Frequency differences for the model with the Planck-Larkin equation minus the model with the Saha equation are plotted. The values of ℓ are indicated as in Figure 1.

Recently, Shibahashi, Noels and Gabriel (1983) have studied the influence of the equation of state on the solar eigenfrequencies and state that the inclusion of the coulomb corrections improves the agreement between theory and observation. Our calculations which explicitly study the influence of this term are in disagreement with their conclusion. We feel that the changes between the Shibahashi and Osaki (1981) frequencies and the Shibahashi, Noels and Gabriel (1983) frequencies are most probably a consequence of other modifications in their numerical procedures such as possibly the interpolation in the Los Alamos Opacity Library.

References

Ebeling, W. and Sandig, R. 1973, Ann. Physik, 28, 289.

Eyring, H., Henderson, D., Stover, B. J., and Eyring, E. M. 1982, **Statical Mechanics and Dynamics**, John Wiley & Sons: New York, p. 548-584.

Grec, G., Fossat, E. and Pomerantz, M. 1983, Solar Physics, 82, 55.

Mayer, J. E. 1958, **Handbuch der Physik**, Springer, Berlin, 12, 123.

Rogers, F. J. 1977, Physics Letters, 61A, 358.

Rogers, F. J. 1984, This Volume.

Rouse, C. A. 1983, Ap. J., 272, 377.

Shibahashi, H., Noels, A. and Gabriel, M. 1983, Astron. and Astroph., 123, 283.

Shibahashi, H., and Osaki, Y. 1981, Publ. Astron. Soc. Japan, 33, 713.

Ulrich, R. K. 1982, Ap. J., 258, 404.

Ulrich, R. K. and Rhodes, E. J., Jr. 1983, Ap. J., 265, 551.